TWEDA User's Manual
(Version 1.0)

by

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0. Introduction

TWEDA (Two-way Exploratory Data Analysis) is an interactive program for the analysis of unreplicated two-way data. The references cited provide the stimulus for and most of the techniques in TWEDA and should be used as necessary to clarify procedures and interpretation.

This manual is intended only as a guide to the operation of TWEDA and not as a text on non-additivity. However, many of the salient ideas of non-additivity are introduced to clarify the options available. TWEDA offers many techniques which were formerly obscure or unavailable and it is this unification of techniques that makes TWEDA unique. For other applications of TWEDA to unreplicated data sets, see Tukey (1965).

As non-additivity is an active area of research, TWEDA is expected to be modified and enlarged periodically. Moreover, bugs in the program may be found requiring changes. The version number on the program header will reflect the current version. The user manual will be updated only when the digit to the left of the decimal is increased. Documentation of any problems (e.g. output etc.) and suggestions for improvements would be greatly appreciated. Send such material to Dr. R. D. Cook, Department of Applied Statistics, 352 Classroom Office Building, University of Minnesota, Saint Paul, MN 55108, (612) 373-0970.

Most of the program was written by L. A. Thibodeau. Important suggestions and technical expertise were contributed by Christopher Bingham and R. Dennis Cook.

The program is written in FORTRAN and should run on CDC CYBER computers with a KRONOS operating system.

<u>Disclaimer</u>. TWEDA has been extensively tested and examined for accuracy, and, to the best of our knowledge, is accurate. However, neither the University of Minnesota nor any of the authors claim any responsibility for any errors that do arise.

1. General Information

1.1 Models and Analysis. Consider a rxc two-way table of response variables y_{ij}, $i=1,2,\ldots,r$, $j=1,2,\ldots,c$, then the additive model is:

$$y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij} \ .$$

Where μ is a grand mean, α_i is a row effect, β_j is a column effect and

$$\sum_i \alpha_i = \sum_j \beta_j = 0 \ .$$

Further the ϵ_{ij} are errors resulting from some spherical distribution with mean zero and unknown finite variance σ^2.

Non-additivity analysis is based on the addition of interaction terms to the additive model. For example, the most common and simplest method of analysis was proposed by Tukey (1949) and results from the addition of an interaction term of the form $\theta\alpha_i\beta_j$. The model with this term added is referred to as Tukey's model:

$$y_{ij} = \mu + \alpha_i + \beta_j + \theta\alpha_i\beta_j + \epsilon_{ij} \ .$$

The term $\theta\alpha_i\beta_j$ has a single degree of freedom associated with it and thus this analysis is referred to as Tukey's one-degree of freedom for non-additivity. Johnson and Graybill (1972) also provided valuable contributions to the analysis of a generalization of this model.

Mandel (1969, 1971) discussed several models for non-additive data. The most general model which we will refer to as Mandel's model is of the form:

$$y_{ij} = \mu + \alpha_i + \beta_j + \lambda_{(1)} w_{i(1)} v_{j(1)} + \lambda_{(2)} w_{i(2)} v_{j(2)} + \cdots + \epsilon_{ij} \ .$$

Let $R = \{r_{ij}\}$ be the rxc residual matrix from the additive model:

$$r_{ij} = y_{ij} - \hat{y}_{ij}$$
$$= y_{ij} - (\hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j).$$

Then the least squares estimators of $\lambda_{(k)}$, $w_{i(k)}$ and $v_{j(k)}$ are $\hat{\lambda}_{(k)}$, $\hat{w}_{i(k)}$ and $\hat{v}_{j(k)}$; where $\hat{\lambda}_{(k)}$ is the k-th eigenvalue of RR' and $\hat{w}'_{(k)} = (\hat{w}_{1(k)}, \hat{w}_{2(k)}, \ldots, \hat{w}_{r(k)})$ and $\hat{v}'_{(k)} = (\hat{v}_{1(k)}, \ldots \hat{v}_{c(k)})$ are the corresponding eigenvectors of $R'R$ and RR' respectively. This model is also referred to as the principal components model for obvious reasons. Two important special cases of this model are the <u>Row-model</u> (row-regression-model):

$$y_{ij} = \mu + \alpha_i + \beta_j + \partial_j \alpha_i + \epsilon_{ij}$$

and the <u>column-model</u> (column-regression-model):

$$y_{ij} = \mu + \alpha_i + \beta_j + \delta_i \beta_j + \epsilon_{ij} .$$

The parameters ∂_j and δ_i are referred to as the column and row slopes, respectively. It can be shown that the terms $\partial_j \alpha_i$ and $\delta_i \beta_j$ are orthogonal and thus the combined or <u>slopes-model</u>:

$$y_{ij} = \mu + \alpha_i + \beta_j + \delta_i \beta_j + \partial_j \alpha_i + \theta \alpha_i \beta_j + \epsilon_{ij}$$

is of special interest.

Initially, concern usually centers on testing one or more of the hypotheses:

Tukey's: $H_0 : \theta = 0$,

Mandel's: $H_0 : \lambda_{(k)} = 0$ for some k ,

Row: $H_0 : \partial_j = 0$ for all j ,

Column: $H_0 : \delta_i = 0$ for all i .

If a hypothesis is rejected then the analyst is usually interested in the reason for the rejection. Discussions of these reasons may be found in Cook (1975), Johnson and Graybill (1972), Mandel (1971) and Tukey (1949 , 1965).

TWEDA provides test statistics for all of the above hypotheses as well as graphical, semi-graphical and numerical devices to illuminate the source of non-additivity.

1.2 Data Files. Data for this program must be on a local KRONOS coded file with any name of seven or less characters beginning with a letter. There is no option for input via TTY. Data is assumed to be formatted in a row by column matrix with the first column line numbers. The term "column" does not imply that the numbers in each row will necessarily align. It does, however, mean that the column entries are in the correct order and each row has the same number of columns. Each number must be separated by blanks or commas and consequently blanks do not denote zero entries. Continuation lines are not allowed. Decimal points and scientific notation (E format) are allowed.

The maximum data set that can presently be analyzed in TWEDA is 20 rows by 20 columns. All data sets must have at least 3 rows and 3 columns.

1.3 Accessing TWEDA. The program is currently available on the MECC/MERITSS CYBER 72 computer at the University of Minnesota. Program access is obtained by typing

```
GET,TWEDA/UN=2051999
-TWEDA
```

from any user number. The data file to be analyzed should be a local file prior to calling TWEDA.

The computer will then respond with:

```
TWEDA--TWO WAY EXPLORATORY DATA ANALYSIS-VERSION 1.0
NAME YOUR DATA FILE
?
```

The data file named must conform to the requirements of Section 1.2. Note, user responses are always terminated by a carriage return.

Large data sets or complicated analyses will often require additional time to complete. For such situations it is recommended that the command

SETTL,176

precede the call to TWEDA.

1.4 Proposed Additions. Tests of several more complicated hypotheses such as: $H_0: v_i = \alpha_i$ for all i, will be added to a future version. Additionally, options for the specification of quantitative values for rows and columns and transformation of the response variables will be forthcoming.

2. Computations

Two distinct forms of calculations are of concern; the sums-of-squares for the additive, row, column, and Tukey's models and the eigenvalue, eigenvector analysis of RR' and $R'R$ for Mandel's model. For ANOVA the slope-model is used to partition the total sums-of-squares. Cook (1975) gives a detailed explanation of the computational method used for each term. Essentially the technique is to regress the estimated row (or column) effects on the residuals from the additive model.

The eigenvalue, eigenvector analysis of RR' and $R'R$ uses a Jacobi threshold algorithm from the IMSL (1975) package. This algorithm has been shown to be extremely accurate and efficient but could be replaced by any algorithm which will compute the eigenvalues and vectors of a non-negative definite matrix.

3. Commands

TWEDA is programmed to provide analysis on demand; that is, only the option demanded will be computed and printed. All options are order independent. The example in Section 4 illustrates most of the commands available. To initiate the analysis response to "NEXT STEP?" with any of the following commands:

3.1 CMDS lists commands with a short explanation of options. See example in Section 4.

3.2 LIST lists the current data matrix. It is recommended that the data be listed upon entry and after each modification (CHNG).

3.3 ANOVA results in the computation and output of an analysis of variance table for the slope-model with a subtable for the additive-model. The F-statistics given are for the appropriate mean square divided by the residual mean square from the slope-model. The slope-model F-statistics are for the hypotheses of no significant regression; $H_0:\partial_j = 0$ for all j in the row-model, $H_0:\delta_i = 0$ for all i in the column-model. The F-statistic corresponding to "TUKEYS" is for the one-degree of freedom for non-additivity test; $H_0:\theta = 0$. Note, the interpretation of "ROW" and "COLUMN" effects in the presence of significant non-additivity may not be straight forward.

3.4 ESTIM prints the maximum likelihood (least squares) estimated coefficients $\hat{\mu}$, $\hat{\alpha}$, and $\hat{\beta}$ from the additive-model. These are also the MLE (LSE) for the other models.

3.5 RESID prints the residual matrix R from the additive-model. Note, these are not studentized residuals. Only residuals from the additive model are available.

3.6 RTABL prints a semi-graphical, symbolic, standardized residual table. The residuals from the additive model in R are standardized by the RMSE from the slope-model. The symbols are determined by the following code:

LOW? < -5.0 < LOW < -3.0 < LO < -2.33 < L < -1.28 < - < 0.0

0.0 < + < 1.28 < H < 2.33 < HI < 3.0 < HIG < 5.0 < HIGH

3.7 EIGAN computes and prints the eigenvalues and eigenvectors for the matrices RR' and $R'R$. These eigenvalues and corresponding vectors are, as noted above, the MLE (LSE) of the parameters in Mandel's model. This command also results in the printing of a statistic "LAMBDA" (Λ) which is the largest eigenvalue divided by the sum of the eigenvalues. This statistic provides a test of the hypothesis, $H_0:\lambda_{(1)} = 0$ (see Johnson and Graybill (1972)). A table of critical values for λ is attached. (Table 1)

3.8 PLT6L prints six line plots of variates as specified in options below. The plotting algorithm is from Andrews and Tukey (1973), from which we quote:

> "The 6-line plot represents numbers in a mixed digital system using digits to the base 4. For numbers between -3 and +3, numbers are simply truncated after one quaternary place. Thus 1·960 (decimal) becomes 1·3311...(quaternary) and then 1·3 (truncated quaternary) which is represented as +ONE 3, where "+ONE" indicates the line on which the character "3" is printed. Numbers between -4¼ and -3 or +3 and +4¼ are treated similarly, using the improper quaternary digits 4, 5, 6, 7, and 8. Thus 3·3 (quaternary) is represented as "+TWO 7" and -4·0 (quaternary) as "-TWO 8". Values above 4¼ are represented as "+TWO 9"; values below =4¼ as "-TWO 9". Where over-printing would otherwise appear only the last character to fall on a specific print position is printed."

An example of a 6-line plot is given in Section 4. The following table may be useful in interpreting the plotted values.

	Plotted Character										
	0	1	2	3	4	5	6	7	8	9	
+TWO	1.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	$+\infty$
+ONE	1.00	1.25	1.50	1.75	2.00						
+	.00	.25	.50	.75	1.00						
-	-.00	-.25	-.50	-.75	-1.00						
-ONE	-1.00	-1.25	-1.50	-1.75	-2.00						
-TWO	-2.00	-2.25	-2.50	-2.75	-3.00	-3.25	-3.50	-3.75	-4.00	-4.25	$-\infty$

Break points for values corresponding to plotted characters in a 6-line plot (e.g., a "2" on the line "-ONE" represents a value less than or equal to -1.50 but greater than -1.75).

The options available are:

Options: Ordinates

RES - Residuals r_{ij} from the additive model may be plotted against the abscissa

TUK - Tukey's test $\hat{\alpha}_i\hat{\beta}_j$

ROW - Estimated row effect $\hat{\alpha}_i$

COL - Estimated column effect $\hat{\beta}_j$

NOR - Normal expected order statistics RANKIT

FIT - Predicted values $\hat{y}_{ij} = \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j$

EIG - Elements of the eigenvector corresponding to the eigenvalue specified against the abcissa:

ROW - Estimated row effect $\hat{\alpha}_i$

COL - Estimated column effect $\hat{\beta}_j$

3.9 PLT30 prints 30 line scatter plots of variates as specified in options above. Residuals are identified by alphabetic characters and are ordered by rows (columns) when plotting against estimated column (row) effects. For example r_{12} would be plotted with the character "A" under the "ROW" option and the character "B" under the "COL" option. See Section 4.

3.10 CHNGE allows for the editing of data and for the replacement of observations by generated values for specified models. Each time CHNGE is called the data is initialized with its original values and all editing or replacement proceeds from these values. No more than three new values may be generated at any time. There is no limit on the number of values edited. The generated values are obtained by numerically minimizing the sums of squares for error, residuals, in the model specified. The model for the values may be specified by;

ADD - Additive-model,

ROW - Row-model,

COL - Column-model,

SLP - Slope-model.

As the generation of K, K = 1,2,3, replacement values requires the solution of K nonlinear equations in K unknowns the convergence to a solution cannot be assured. Care must be taken to insure that

1) The answers are reasonable.
2) The SSE for the model specified is, in fact, less than the original SSE.

If any of these conditions are not satisfied, the method of solution is diverging and may be remedied by using better starting values for the replacement values. Plots of residuals versus estimated row or column effects may be helpful.

The replacement values generated will probably be good to 4 or 5 significant digits if the starting values are close. To check the generated values call CHNG and use the generated values as starting values. Note, if CHNG results in the response---FAILURE TO CONVERGE--- then the values at the last iteration are substituted in the data matrix.

3.11 RENEW initializes the data matrix to its original value.

3.12 PARTI allows the selection of a subtable of the original table for analysis. Note, each time PARTI is used the data are restored to their original values.

3.13 END terminates the program.

4. EXAMPLE: The following five pages of sample output are from a preliminary analysis of data on barley yields for six varieties at five experimental stations in Minnesota, ref. Yates (1970) pp. 119-146.

```
 OLD,LTYD/UN=2051950
/LNH
00100 80.85 93.85 100.05 98.45 91.25
00110 123.5 128.75 131.45 169.6 126.9
00120 92.7 91.2 97.4 135.6 109.6
00130 109.35 91.65 110.1 133.15 100.25
00140 82.65 69.45 82.9 75.6 92.2
00150 77.3 71.9 73.15 96.8 95.05
/GET,TWEDA/UN=2051999
/-TWEDA

  TWEDA -- TWO WAY EXPLORATORY DATA ANALYSIS -- VERSION 1.0

 NAME YOUR DATA FILE
? LTYD
 ENTER TOTAL ROW AND COLUMN DIMENSIONS   ? 6 5


 IF YOU NEED HELP AT ANY TIME RESPOND TO "NEXT STEP?" WITH "CMNDS".
 NEXT STEP
? LIST
    80.850      93.850      100.05      98.450      91.250
    123.50      128.75      131.45      169.60      126.90
    92.700      91.200      97.400      135.60      109.60
    109.35      91.650      110.10      133.15      100.25
    82.650      69.450      82.900      75.600      92.200
    77.300      71.900      73.150      96.800      95.050
 NEXT STEP
? ANOVA
                 TWO-WAY ANALYSIS OF VARIANCE
   SOURCE       D.F.    SS              MS                F
   ------------------------------------------------------------
   ROWS          5     10610.          2122.0            41.679
   COLUMNS       4     2655.2          663.79            13.038
   RES(ADD)     20     2217.1          110.86
   ------------------------------------------------------------
      COL-MODEL     4     447.19          111.80            2.196
      ROW-MODEL     3     564.00          188.00            3.693
      TUKEYS        1     594.96          594.96           11.686
      RESIDUAL     12     610.96          50.913
   ------------------------------------------------------------
   ------------------------------------------------------------
   TOTAL        29     15482.


   GRAND MEAN=  101.09


 NEXT STEP
?
```

```
EIGAN

EIGENVALUE       EIGENVECTOR (COLS)
 18.836        -.47329    -.53076     .63766     .81816E-01 .28458

 287.74         .70910    -.56280     .22037    -.35396E-02-.36312

 458.79         .16785    -.43755    -.55753     .16123     .66599

 1451.7         .21214     .10049     .18437    -.87596     .37895


 LAMDA .65479

EIGENVALUE       EIGENVECTOR (ROWS)
 18.836         .16036    -.46649     .70637     .42283E-01 .59826E-01
              -.50235
 287.74         .39900     .14412     .25071    -.79602    -.24742
               .24960
 458.79        -.68414    -.23612     .38202    -.13643     .13233
               .54234
 1451.7        -.27775     .52197     .34766     .22898    -.67520
              -.14565
NEXT STEP
? RTABL

          -      H      +      L      -
          -      +      -      H      L
          -      -      -      H      +
          +      -      +      +      L
          +      -      +     LOW     H
          +      -      -      -      H
NEXT STEP
? RESID
  -5.3433     10.915     9.0733    -11.552     -3.0933
  -5.8433     2.6650    -2.6767     16.448     -10.593
  -5.9033    -4.1450    -5.9867     13.188      2.8467
   7.1467    -7.2950     3.1133     7.1383    -10.103
   8.7867    -1.1550     4.2533    -22.072      10.187
   1.1567    -.98500    -7.7767    -3.1517      10.757
NEXT STEP
?
```

```
  PLT30
 VERT(RES,EIG)
? RES
 HORZ(TUK,ROW,COL,NOR,FIT)
? ROW



CORRELATION COEFFICIENT =    .00000
 16.45    *                                                         D
          +
          +                         D
          +
          +E E         B
          +A           C
          +
          +                             D
          +
          +C
          +                         E   C                           B
          +  A
          +
          +B B
          +  D         E                                            C
          +                         B
          +            A            C                               A
          +                             B
          +  C
          +
          +                             E                           E
          +            D
          +
          +
          +
          +
          +
          +
          +D
-23.40    *+++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
     -21.49          Y-INCRE= 1.33287    /X-INCRE.= .958204        34.95

 NEXT STEP
? PARTI
 ENTER ROW AND COLUMN DIMENSIONS FOR SUBTABLE
? 6 4
 ENTER ROW INDICES
? 1 2 3 4 5 6
 ENTER COL INDICES
? 1 2 3 5
 NEXT STEP
?
```

```
 LIST
   80.850      93.850      100.05      91.250
   123.50      128.75      131.45      126.90
   92.700      91.200      97.400      109.60
   109.35      91.650      110.10      100.25
   82.650      69.450      82.900      92.200
   77.300      71.900      73.150      95.050
 NEXT STEP
? ANOVA
                  TWO-WAY ANALYSIS OF VARIANCE
  SOURCE       D.F.    SS              MS                F
  ------------------------------------------------------------
  ROWS          5     6186.5         1237.3           18.740
  COLUMNS       3     459.11         153.04            2.318
  RES(ADD)     15     809.65         53.976
  ------------------------------------------------------------
     COL-MODEL     4     73.827         18.457             .280
     ROW-MODEL     2     98.653         49.327             .747
     TUKEYS        1     108.97         108.97            1.650
     RESIDUAL      8     528.19         66.024
  ------------------------------------------------------------
  ------------------------------------------------------------
  TOTAL        23     7455.2


  GRAND MEAN=  96.810


 NEXT STEP
? EIGAN


  EIGENVALUE        EIGENVECTOR (COLS)
   29.986       -.46053     -.49855      .67884      .28024
   287.76        .70973     -.56427      .21650     -.36197
   491.90        .18494     -.42785     -.49224      .73515


   LAMDA .60755


  EIGENVALUE        EIGENVECTOR (ROWS)
   29.986       -.42213      .62516     -.27738      .67974E-01-.41479
                 .42117
   287.76        .40482      .14125      .24626     -.79698     -.24296
                 .24762
   491.90        .55902      .39186     -.25796      .20367     -.33880
                -.55779
 NEXT STEP
?
```

```
  PLT6L
 VERT(RES,EIG)
? RES
 HORZ(TUK,ROW,COL,NOR,FIT)
? NOR



 W-STATISTIC =     .95616
+TWO
+ONE                                    0 0 0 1   2    2
   +                              002 33
   -                    3 11 1100
-ONE 1    1   1   0 0 0
-TWO
       ....^.........^.........0.........^.........^....
 LOBOUNDS=  -1.9469    /UPBOUNDS=   1.9469    /INCR.=    .80000
 NEXT STEP
? RENEW
 NEXT STEP
? CHNGE
 ENTER ROW AND COLUMN? 5 4
 DO YOU HAVE A REPLACEMENT VALUE, YES OR NO.        ? NO
 IS THAT ALL, YES OR NO.        ? YES
 WHICH MODEL:(ADD,ROW,COL,OR SLP),       ? ADD
 NEXT STEP
? LIST
    80.850      93.850      100.05      98.450      91.250
    123.50      128.75      131.45      169.60      126.90
    92.700      91.200      97.400      135.60      109.60
    109.35      91.650      110.10      133.15      100.25
    82.650      69.450      82.900      108.69      92.200
    77.300      71.900      73.150      96.800      95.050
 NEXT STEP
? ANOVA
                TWO-WAY ANALYSIS OF VARIANCE
  SOURCE        D.F.     SS              MS              F
  ------------------------------------------------------------
  ROWS            5     9433.9         1886.8           36.938
  COLUMNS         4     3933.7         983.43           19.253
  RES(ADD)       19     1486.4         74.318
  ------------------------------------------------------------
     COL-MODEL     4      385.61         96.402             1.887
     ROW-MODEL     3      319.01         106.34             2.082
     TUKEYS        1      219.87         219.87             4.304
     RESIDUAL     11      561.88         51.080
  ------------------------------------------------------------
  ------------------------------------------------------------
  TOTAL          28     14854.


  GRAND MEAN=  102.19


 NEXT STEP
?
```

TABLE 1

I	J: 3	4	5	6	7	8	10	12	16	20
					Upper 10%					
3	.9975a									
4	.9743a	.8349								
5	.9429a	.8458a	.8021							
6	.9135a	.8130	.6975	.6398						
7	.8879a	.7631a	.6548	.6358	.5687					
8	.8660a	.7435	.6487	.5725	.5462	.5098				
10	.8308a	.6749	.6057	.5570	.4972	.4489	.3982			
12	.8037a	.6594	.5695	.5001	.4563	.4289	.3830	.3506		
16	.7647a	.6022	.5131	.4550	.4137	.3815	.3268	.2952	.2562	
20	.7376a	.5737	.4843	.4329	.3778	.3502	.3023	.2765	.2326	.2049
32	.6886a	.5161	.4306	.3740	.3329	.2992	.2556	.2257	.1881	.1654
50	.6512a	.4887	.3978	.3362	.2913	.2624	.2226	.1960	.1598	.1386
100	.6071a	.4421	.3490	.2942	.2533	.2255	.1850	.1595	.1279	.1001
					Upper 5%					
3	.9994a									
4	.9873a	.8567								
5	.9648a	.8811a	.8407							
6	.9406a	.8505	.7294	.6681						
7	.9168a	.8003a	.6823	.6703	.5957					
8	.8974a	.7811	.6815	.5985	.5733	.5345				
10	.8630a	.7043	.6361	.5901	.5096	.4680	.4143			
12	.8357a	.6936	.5979	.5242	.4774	.4501	.4016	.3665		
16	.7950a	.6295	.5356	.4760	.4227	.3991	.3390	.3064	.2656	
20	.7661a	.6290	.5054	.4542	.3932	.3652	.3139	.2876	.2408	.2117
32	.7127a	.5349	.4469	.3894	.3454	.3105	.2644	.2335	.1938	.1702
50	.6713a	.5978	.4127	.3482	.3002	.2706	.2296	.2020	.1643	.1423
100	.6218a	.4610	.3583	.3024	.2595	.2311	.1892	.1629	.1306	.1116
					Upper 1%					
3	.99997a									
4	.9975a	.8930								
5	.9883a	.9303a	.9004							
6	.9743a	.9082	.7825	.7194						
7	.9587a	.8619a	.7325	.7325	.6457					
8	.9429a	.8446	.7407	.6470	.6243	.5809				
10	.9135a	.7575	.6924	.6516	.5523	.5044	.4452			
12	.8879a	.7411	.6514	.5702	.5170	.4911	.4372	.3969		
16	.8472a	.6256	.5788	.5167	.4560	.4331	.3628	.3372	.2837	
20	.8164a	.5966	.5462	.4955	.4229	.3945	.3364	.3095	.2567	.2249
32	.7571a	.5367	.4788	.4198	.3700	.3326	.2818	.2485	.2049	.1796
50	.7089a	.5043	.4423	.3722	.3178	.2864	.2430	.2139	.1731	.1495
100	.6498a	.4463	.3771	.3189	.2720	.2421	.1977	.1698	.1359	.1159

[a]Indicates exact critical point. Remaining values were obtained by Monte Carlo.

5. REFERENCES.

[1] Andrews, D.F. and J.W. Tukey (1973). Teletypwriter plots for data analysis can be fast: 6-line plots including probability plots. Applied Statistics 22, 192-203.

[2] Anscombe, F.J. and John W. Tukey (1963). The examination and analysis of residuals, Technometrics 5, 141-160.

[3] Cook, R.D.(1975), Analysis of interactions, Unpublished manuscript.

[4] Graybill, R.A. (1969). Introduction to Matrices with Applications in Statistics, Belmont, CA: Wadsworth Publishing Co.

[5] Hegemann, V. and D.E. Johnson (1976). On analyzing two-way AoV data with interaction. Technometrics, 18, 273-282.

[6] Hegemann, V. and D.E. Johnson (1976). The power of two tests for non-additivity. JASA, 71, 945-948.

[7] Johnson, D.E. and F.A. Graybill (1972). An analysis of a two-way model with interaction and no replication, JASA, 67, 862-868.

[8] Johnson, D.E. (1976). Some new multiple comparison procedures for the two-way AoV model with interaction. Biometrics, 32, 929-934.

[9] Mandel, John (1960). The partitioning of interaction in analysis of variance, Journal of Research of the Bureau of Standards - B. Mathematical Sciences, 73B, 309-328.

[10] Mandel, John (1970). The distribution of eignevalues of covariance matrices of residuals in analysis of variance, Journal of Research of the Bureau of Standards - B. Mathematical Sciences, 74B, 149-154.

[11] Mandel, John (1971). A new analysis of variance model for non-additive data, Technometrics, 13, 1-18.

[12] Tukey, John W. (1949). One degree of freedom for non-additivity, Biometrics, 5, 232-242.

[13] Tukey, John W. (1965). Introduction to Exploratory Data Analysis, Limited edition, experimental version, Princeton University.

[14] Weisberg, S. and C. Bingham (1975). An Approximate Analysis of Variance Test for Non-normality Suitable for Machine Calculation. Technometrics, 17, 133-134.

[15] Yates, F. (1970). Experimental Design-Selected Papers, Griffin, London.